

# Package ‘proporz’

October 10, 2025

**Type** Package

**Title** Proportional Apportionment

**Version** 1.5.2

**Description** Calculate seat apportionment for legislative bodies with various methods. The algorithms include divisor or highest averages methods (e.g. Jefferson, Webster or Adams), largest remainder methods and biproportional apportionment.  
Gaffke, N. & Pukelsheim, F. (2008) <[doi:10.1016/j.mathsocsci.2008.01.004](https://doi.org/10.1016/j.mathsocsci.2008.01.004)>  
Oelbermann, K. F. (2016) <[doi:10.1016/j.mathsocsci.2016.02.003](https://doi.org/10.1016/j.mathsocsci.2016.02.003)>.

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**URL** <https://polettif.github.io/proporz/>,  
<https://github.com/polettif/proporz>

**BugReports** <https://github.com/polettif/proporz/issues>

**VignetteBuilder** knitr

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## Contents

|                        |   |
|------------------------|---|
| apply_quorum . . . . . | 2 |
| biproporz . . . . .    | 3 |

|                                       |           |
|---------------------------------------|-----------|
| ceil_at . . . . .                     | 5         |
| district_winner_matrix . . . . .      | 6         |
| divisor_methods . . . . .             | 7         |
| finland2019 . . . . .                 | 8         |
| get_divisors . . . . .                | 9         |
| highest_averages_method . . . . .     | 9         |
| largest_remainder_method . . . . .    | 10        |
| lower_apportionment . . . . .         | 11        |
| pivot_to_matrix . . . . .             | 13        |
| proporz . . . . .                     | 14        |
| pukelsheim . . . . .                  | 15        |
| quorum_functions . . . . .            | 17        |
| reached_quorum_any_district . . . . . | 19        |
| reached_quorum_total . . . . .        | 19        |
| run_app . . . . .                     | 20        |
| upper_apportionment . . . . .         | 21        |
| uri2020 . . . . .                     | 22        |
| weight_votes_matrix . . . . .         | 23        |
| zug2018 . . . . .                     | 23        |
| <b>Index</b>                          | <b>25</b> |

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|              |                                               |
|--------------|-----------------------------------------------|
| apply_quorum | <i>Apply quorum to votes vector or matrix</i> |
|--------------|-----------------------------------------------|

---

**Description**

This quorum calculation implementation is called within `proporz()`, `biproporz()` and related functions. Generally, there’s no need to call `apply_quorum` directly.

**Usage**

```
apply_quorum(votes, quorum)
```

**Arguments**

- |        |                                                                                                                                                                                                                                                                                                                                                                         |
|--------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| votes  | votes vector or votes matrix                                                                                                                                                                                                                                                                                                                                            |
| quorum | Depending on votes: <ul style="list-style-type: none"><li>• For a vector: Vote threshold a party must reach. Used as fraction of total votes if less than 1 otherwise as number of votes.</li><li>• For a matrix: List of quorum functions (created with <code>quorum_functions</code>) or a logical vector with the same length as the number of votes rows.</li></ul> |

**Value**

Vector or matrix with same dimension as votes. Parties that failed to reach the specified quorum have their votes set to zero.

**See Also**

[quorum\\_functions](#) for more matrix examples.

**Examples**

```
# vector
(votes = c(81, 9, 10))

apply_quorum(votes, 10)

apply_quorum(votes, .11)

# matrix
(votes_matrix = matrix(c(91, 9, 199, 1), nrow = 2))

apply_quorum(votes_matrix, quorum_all(total = 0.1))

apply_quorum(votes_matrix, c(FALSE, TRUE))
```

---

biproporz

*Biproportional apportionment*


---

**Description**

Method to proportionally allocate seats among parties (or lists) and districts (or entities, regions), thus bi-proportional.

**Usage**

```
biproporz(
  votes_matrix,
  district_seats,
  quorum,
  weight_votes = TRUE,
  method = "round",
  ...
)
```

**Arguments**

**votes\_matrix**     Vote count matrix with votes by party in rows and votes by district in columns.

**district\_seats**   Vector defining the number of seats per district. Must be the same length as `ncol(votes_matrix)`. Values are name-matched to `votes_matrix` columns if both are named. If the number of seats per district should be calculated according to the number of votes (not the general use case), a single number for the total number of seats can be used.

|              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
|--------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| quorum       | Optional list of functions which take the <code>votes_matrix</code> and return a logical vector that denotes for each party/row whether they reached the quorum (i.e. are eligible for seats). The easiest way to do this is via <code>quorum_any()</code> or <code>quorum_all()</code> , see examples. Alternatively you can pass a precalculated logical vector. No quorum is applied if this parameter is missing or NULL.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
| weight_votes | By default (TRUE) it is assumed that each voter in a district has as many votes as there are seats in a district. Thus, votes are weighted according to the number of available district seats with <code>weight_votes_matrix()</code> . Set to FALSE if the argument <code>votes_matrix</code> shows the number of <i>voters</i> (e.g. because they can only cast one vote for one party).                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| method       | Defines which method is used to assign seats. The following methods are recommended: <ul style="list-style-type: none"> <li>• round: Uses the Sainte-Laguë/Webster method (rounding half up) for the upper and lower apportionment which is the standard for biproportional apportionment and the only method guaranteed to terminate.</li> <li>• wto: "winner take one" works like "round" with a condition that the party that got the most votes in a district must get <i>at least</i> one seat ('Majorzbedingung') in said district. This only applies if they got enough seats in the upper apportionment (which uses the Sainte-Laguë/Webster method). See <code>lower_apportionment()</code> for more details.</li> </ul> <p>It is also possible to use any divisor method name listed in <code>proporz()</code>. If you want to use a different method for the upper and lower apportionment, provide a list with two entries.</p> |
| ...          | ignored (available for backwards compatibility)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |

## Details

Each party nominates a candidate list for every district. The voters vote for the parties of their district. The seat allocation is calculated in two steps:

1. In the so called `upper apportionment` the number of seats for each party (over all districts) is determined. Normally, the number of seats for each region are defined before the election and are independent of the vote counts.
2. In the so called `lower apportionment` the seats are distributed to the regional party list respecting the results from the upper apportionment.

Parties failing to reach quorums cannot get seats. This function does not handle seat assignment to candidates.

## Value

Matrix with the same dimension as `votes_matrix` containing the number of seats with the row and column divisors stored in attributes (hidden from print, see `get_divisors()`).

## Note

The iterative process in the lower apportionment is only guaranteed to terminate with the default Sainte-Laguë/Webster method.

## References

Gaffke, Norbert; Pukelsheim, Friedrich (2008): Divisor methods for proportional representation systems: An optimization approach to vector and matrix apportionment problems. *Mathematical Social Sciences*, 56 (2), 166-184.

## See Also

[pukelsheim\(\)](#) for biproportional apportionment with `data.frames` as inputs.

## Examples

```
votes_matrix = uri2020$votes_matrix
district_seats = uri2020$seats_vector

biproporz(votes_matrix, district_seats)

# apply quorum (high values for illustrative purposes)
biproporz(votes_matrix, district_seats,
          quorum_all(any_district = 0.1, total = 0.25))
```

---

|         |                                            |
|---------|--------------------------------------------|
| ceil_at | <i>Rounding with predefined thresholds</i> |
|---------|--------------------------------------------|

---

## Description

Round  $x$  up to  $\text{ceiling}(x)$  if  $x - \text{floor}(x) \geq \text{threshold}$ , otherwise round down to  $\text{floor}(x)$ .

## Usage

```
ceil_at(x, threshold)
```

## Arguments

|                        |                                                                                             |
|------------------------|---------------------------------------------------------------------------------------------|
| <code>x</code>         | numeric vector or matrix $\geq 0$ (NaN is not supported)                                    |
| <code>threshold</code> | threshold in $[0,1]$ or "harmonic"/"geometric" to use harmonic or geometric mean thresholds |

## Value

the rounded vector or matrix

## Examples

```
ceil_at(c(0.5, 1.5, 2.49, 2.5, 2.51), 0.5)
# compare to
round(c(0.5, 1.5, 2.49, 2.5, 2.51))

ceil_at(c(1.45, 2.45, 3.45), 0) # like floor()
ceil_at(c(1.45, 2.45, 3.45, 0.2), "geometric")
```

---

district\_winner\_matrix

*Find which party has the most votes in a district*


---

## Description

Create a logical matrix that shows whether a party got the most votes in a district or not.

## Usage

```
district_winner_matrix(votes_matrix, district_seats = 1L)
```

## Arguments

**votes\_matrix**     Vote count matrix with votes by party in rows and votes by district in columns.

**district\_seats**   Vector defining the number of seats per district. Must be the same length as `ncol(votes_matrix)`. Values are name-matched to `votes_matrix` columns if both are named. If a single value is supplied (like 1 as default), it is used as the number of seats for every district.

## Details

If two or more parties are tied and there are not enough seats for each tied party, the matrix value is NA.

## Value

logical matrix with the same dimensions and names as `votes_matrix`

## Examples

```
(vm = matrix(c(60,30,0,20,10,30), nrow = 3, dimnames = list(1:3, c("A", "B"))))

district_winner_matrix(vm)

# NA values if parties are tied (here in district B)
vm[1,2] <- 30
district_winner_matrix(vm)

# No NA values for tied parties if enough seats are available
district_winner_matrix(vm, c(1, 2))
```

---

|                 |                        |
|-----------------|------------------------|
| divisor_methods | <i>Divisor methods</i> |
|-----------------|------------------------|

---

## Description

Functions to directly apply divisor apportionment methods instead of calling `proporz()` with a method parameter. All divisor functions call `highest_averages_method()` with a different sequence of divisors.

## Usage

```
divisor_round(votes, n_seats, quorum = 0)
```

```
divisor_floor(votes, n_seats, quorum = 0)
```

```
divisor_harmonic(votes, n_seats, quorum = 0)
```

```
divisor_geometric(votes, n_seats, quorum = 0)
```

```
divisor_ceiling(votes, n_seats, quorum = 0)
```

## Arguments

|         |                                                                                                                        |
|---------|------------------------------------------------------------------------------------------------------------------------|
| votes   | numeric vector with number of votes for each party                                                                     |
| n_seats | total number of seats                                                                                                  |
| quorum  | Vote threshold a party must reach. Used as fraction of total votes within if less than 1 otherwise as number of votes. |

## Details

Divisor methods are known under different names:

- d'hondt, jefferson, hagenbach-bischoff: `divisor_floor()`
- sainte-lague, webster: `divisor_round()`
- adams: `divisor_ceiling()`
- dean: `divisor_harmonic()`
- huntington-hill, hill-huntington: `divisor_geometric()`

## Value

The number of seats per party as a vector

## See Also

`proporz()`, `highest_averages_method()`

**Examples**

```
votes = c("Party A" = 690, "Party B" = 400,  
          "Party C" = 250, "Party D" = 120)  
  
divisor_round(votes, 10)  
  
divisor_floor(votes, 10)  
  
divisor_ceiling(votes, 10)  
  
divisor_ceiling(votes, 5)  
  
divisor_geometric(votes, 10, quorum = 0.05)  
  
divisor_harmonic(votes, 10)
```

---

finland2019

*Finnish Parliamentary Elections Data (2019)*

---

**Description**

Example data from the 2019 Finnish parliamentary elections. The data has been cleaned up and only contains information relevant for this package.

**Usage**

```
finland2019
```

**Format**

List containing two data.frames:

- votes\_df containing the number of votes for each party and district. 229 rows, 3 columns (party\_name, district\_name, votes)
- district\_seats\_df with the number of seats per district. 12 rows, 2 columns (district\_name, seats)

**Source**

[https://tulospalvelu.vaalit.fi/EKV-2019/en/ladattavat\\_tiedostot.html](https://tulospalvelu.vaalit.fi/EKV-2019/en/ladattavat_tiedostot.html)

**Examples**

```
finland2019$district_seats_df  
  
head(finland2019$votes_df)
```

---

get\_divisors

*Get district and party divisors from biproporz result*


---

### Description

Show the district and party divisors used to assign seats. This method provides easier access to divisors stored in `attributes(...)$divisors`.

### Usage

```
get_divisors(biproporz_result)
```

### Arguments

biproporz\_result

a matrix created by `biproporz()` or a data.frame created by `pukelsheim()`

### Value

The district and party divisors (named "districts" and "parties") in a list, each as a vector

### Examples

```
seats_matrix = biproporz(uri2020$votes_matrix, uri2020$seats_vector)
get_divisors(seats_matrix)

seats_df = pukelsheim(pivot_to_df(uri2020$votes_matrix),
                      data.frame(names(uri2020$seats_vector), uri2020$seats_vector))
get_divisors(seats_df)

# summary() also prints the divisors for a biproporz matrix
summary(seats_matrix)
```

---

highest\_averages\_method

*Highest averages method*


---

### Description

Allocate seats proportionally for [divisor methods](#).

### Usage

```
highest_averages_method(votes, n_seats, divisors)
```

**Arguments**

|          |                                                                                                                                                             |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------|
| votes    | numeric vector with number of votes for each party                                                                                                          |
| n_seats  | total number of seats                                                                                                                                       |
| divisors | sequence of divisors (length equal to the number of seats). If it is a single number (e.g. 0.5), a sequence is generated starting with it, increasing by 1. |

**Details**

The highest averages method requires the number of votes for each party to be divided successively by a series of divisors. This produces a table of quotients, or averages, with a row for each divisor and a column for each party. The *n*th seat is allocated to the party whose column contains the *n*th largest entry in this table, up to the total number of seats available. ([Wikipedia](#))

**Value**

The number of seats per party as a vector

**Examples**

```
highest_averages_method(c(5200, 1700, 3100), 15, 0.5)

highest_averages_method(votes = c(50, 0, 30), n_seats = 3,
                        divisors = c(0, 1.3333, 2.4))
```

---

largest\_remainder\_method

*Largest remainder method*

---

**Description**

Allocate seats based on the largest fractional remainder. The largest remainder method is also known as Hamilton, Hare-Niemeyer or Vinton method.

**Usage**

```
largest_remainder_method(votes, n_seats, quorum = 0)
```

**Arguments**

|         |                                                                                                                        |
|---------|------------------------------------------------------------------------------------------------------------------------|
| votes   | numeric vector with number of votes for each party                                                                     |
| n_seats | total number of seats                                                                                                  |
| quorum  | Vote threshold a party must reach. Used as fraction of total votes within if less than 1 otherwise as number of votes. |

**Details**

The numbers of votes for each party is divided by a quota representing the number of votes required for a seat. Then, each party receives the rounded down quota value as seats. The remaining seats are given to the party with the largest remainder until all seats have been distributed.

**Value**

The number of seats per party as a vector

**Note**

Only the quota  $\text{total votes} / \text{total seats}$  (which is used by the aforementioned methods) is implemented.

**See Also**

[proporz\(\)](#)

**Examples**

```
votes = c(47000, 16000, 15800, 12000, 6100, 3100)
largest_remainder_method(votes, 10)
```

---

|                     |                            |
|---------------------|----------------------------|
| lower_apportionment | <i>Lower apportionment</i> |
|---------------------|----------------------------|

---

**Description**

In the second biproportional apportionment step, party and district divisors are calculated such that the row and column sums of the resulting seats matrix satisfy the constraints given by the upper apportionment.

**Usage**

```
lower_apportionment(votes_matrix, seats_cols, seats_rows, method = "round")
```

**Arguments**

- |              |                                                                                                                                                                                                                                                                                           |
|--------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| votes_matrix | matrix with votes by party in rows and votes by district in columns.                                                                                                                                                                                                                      |
| seats_cols   | number of seats per column (districts/regions), predetermined or calculated with <a href="#">upper_apportionment()</a> .                                                                                                                                                                  |
| seats_rows   | number of seats per row (parties/lists), calculated with <a href="#">upper_apportionment()</a> .                                                                                                                                                                                          |
| method       | Apportion method that defines how seats are assigned. The following methods are supported: <ul style="list-style-type: none"> <li>round: The default Sainte-Laguë/Webster method is the standard for biproportional apportionment and the only method guaranteed to terminate.</li> </ul> |

- wto: "winner take one" works like round with a condition that the party that got the most votes in a district must get *at least* one seat ('Majorzbedingung', also called 'strongest party constrained' rule (SPC)). votes\_matrix must have row and column names to use this method. A district winner can only get a seat if they are entitled to one from the upper apportionment (seats\_rows). The condition does not apply in a district if two or more parties have the same number of votes and there are not enough seats for these parties. A warning is issued in this case. Modify the votes matrix to explicitly break ties.
- You can provide a custom function that rounds a matrix (i.e. the votes\_matrix divided by party and district divisors) without further parameters.
- It is possible to use any divisor method name listed in `proporz()`.

### Details

The result is obtained by an iterative process ('Alternate Scaling Algorithm', see Reference). Initially, for each district a divisor is chosen using the highest averages method for the votes allocated to each regional party list in this region. For each party a party divisor is initialized with 1.

Effectively, the objective of the iterative process is to modify the regional divisors and party divisors so that the number of seats in each regional party list equals the number of their votes divided by both the regional and the party divisors.

The following two correction steps are executed until this objective is satisfied:

- modify the party divisors such that the apportionment within each party is correct with the chosen rounding method,
- modify the regional divisors such that the apportionment within the region is correct with the chosen rounding method.

### Value

A seat matrix with district (columns) and party (rows) divisors stored in attributes.

### Note

If the maximum number of optimization iterations is reached, an error is thrown since no solution can be found. You can overwrite the default (1000) with `options(proporz_max_iterations = ...)` but it is very likely that the result is undefined given the structure of the input parameters.

### References

Oelbermann, K. F. (2016): Alternate scaling algorithm for biproportional divisor methods. *Mathematical Social Sciences*, 80, 25-32.

### See Also

`biproporz()`, `upper_apportionment()`, `district_winner_matrix()`

**Examples**

```

votes_matrix = matrix(c(123,912,312,45,714,255,815,414,215), nrow = 3)
district_seats = c(7,5,8)
party_seats = c(5,11,4)

lower_apportionment(votes_matrix, district_seats, party_seats)

# using "winner take one"
vm = matrix(c(200,100,10,11), 2,
            dimnames = list(c("Party A", "Party B"), c("I", "II")))
district_seats = setNames(c(2,1), colnames(vm))
ua = upper_apportionment(vm, district_seats)

lower_apportionment(vm, ua$district, ua$party, method = "wto")

# compare to standard method
lower_apportionment(vm, ua$district, ua$party, method = "round")

```

---

|                 |                                                            |
|-----------------|------------------------------------------------------------|
| pivot_to_matrix | <i>Pivot long data.frame to wide matrix and vice versa</i> |
|-----------------|------------------------------------------------------------|

---

**Description**

Create a matrix in 'wide' format from a data.frame with 3 columns with `pivot_to_matrix` or create a data.frame in long format from a matrix with `pivot_to_df`.

**Usage**

```

pivot_to_matrix(df_long)

pivot_to_df(matrix_wide, value_colname = "values")

```

**Arguments**

|               |                                                           |
|---------------|-----------------------------------------------------------|
| df_long       | data.frame in long format with exactly 3 columns          |
| matrix_wide   | matrix in wide format                                     |
| value_colname | name for the new value column in the resulting data.frame |

**Details**

These pivot functions are used to prepare data for `biproporz()` in `pukelsheim()`. They are not supposed to cover general use cases or provide customization. They mainly exist because reshape is hard to handle and the package should have no dependencies.

**Value**

A data.frame with 3 columns or a matrix. Note that the results are sorted by the first and second column (data.frame) or row/column names (matrix).

**Examples**

```
# From data.frame to matrix
df = data.frame(party = c("A", "A", "A", "B", "B", "B"),
                region = c("III", "II", "I", "I", "II", "III"),
                seats = c(5L, 3L, 1L, 2L, 4L, 6L))
pivot_to_matrix(df)

# from matrix to data.frame
mtrx = matrix(1:6, nrow = 2)
pivot_to_df(mtrx)

# from matrix to data.frame using dimnames
dimnames(mtrx) <- list(party = c("A", "B"), region = c("I", "II", "III"))
pivot_to_df(mtrx, "seats")

# Note that pivot results are sorted
pivot_to_df(pivot_to_matrix(df)) == df[order(df[[1]], df[[2]]),]
```

---

|         |                                   |
|---------|-----------------------------------|
| proporz | <i>Proportional apportionment</i> |
|---------|-----------------------------------|

---

**Description**

Calculate seat apportionment for legislative bodies.

**Usage**

```
proporz(votes, n_seats, method, quorum = 0)
```

**Arguments**

|         |                                                                                                                        |
|---------|------------------------------------------------------------------------------------------------------------------------|
| votes   | numeric vector with number of votes for each party                                                                     |
| n_seats | total number of seats                                                                                                  |
| method  | Apportionment method to use, as character. Not case sensitive. See details.                                            |
| quorum  | Vote threshold a party must reach. Used as fraction of total votes within if less than 1 otherwise as number of votes. |

## Details

The following methods are available:

- d'hondt, jefferson, hagenbach-bischoff, floor: `divisor_floor()`
- sainte-lague, webster, round: `divisor_round()`
- adams, ceiling: `divisor_ceiling()`
- dean, harmonic: `divisor_harmonic()`
- huntington-hill, hill-huntington, geometric: `divisor_geometric()`
- hare-niemeyer, hamilton, vinton, largest\_remainder\_method: `largest_remainder_method()`

## Value

The number of seats per party as a vector

## Note

Seats can also be apportioned among regions instead of parties. The parameter votes is then normally used with census data (e.g. population counts).

## Examples

```
votes = c("Party A" = 651, "Party B" = 349, "Party C" = 50)

proporz(votes, 10, "sainte-lague")

proporz(votes, 10, "hill-huntington")

proporz(votes, 10, "hill-huntington", quorum = 0.05)

proporz(votes, 10, "jefferson", quorum = 70)
```

## Description

Method to proportionally allocate seats among parties/lists and districts/regions/entities ('Doppelter Pukelsheim').

**Usage**

```
pukelsheim(
  votes_df,
  district_seats_df,
  quorum,
  new_seats_col = "seats",
  weight_votes = TRUE,
  winner_take_one = FALSE,
  ...
)
```

**Arguments**

|                                |                                                                                                                                                                                                                                                                                                                                                                                                                               |
|--------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>votes_df</code>          | data.frame (long format) with 3 columns (actual colnames can differ): <ol style="list-style-type: none"> <li>1. party id/name (character)</li> <li>2. district id/name (character)</li> <li>3. votes (numeric)</li> </ol>                                                                                                                                                                                                     |
| <code>district_seats_df</code> | data.frame with 2 columns (actual colnames can differ): <ol style="list-style-type: none"> <li>1. district id/name (character)</li> <li>2. number of seats for a district (numeric)</li> </ol>                                                                                                                                                                                                                                |
| <code>quorum</code>            | Optional list of functions which take the <code>votes_matrix</code> and return a logical vector that denotes for each party/row whether they reached the quorum (i.e. are eligible for seats). The easiest way to do this is via <code>quorum_any()</code> or <code>quorum_all()</code> , see examples. Alternatively you can pass a precalculated logical vector. No quorum is applied if this parameter is missing or NULL. |
| <code>new_seats_col</code>     | name of the new column                                                                                                                                                                                                                                                                                                                                                                                                        |
| <code>weight_votes</code>      | By default (TRUE) it is assumed that each voter in a district has as many votes as there are seats in a district. Set to FALSE if <code>votes_df</code> shows the number of <i>voters</i> (e.g. because they can only vote for one party).                                                                                                                                                                                    |
| <code>winner_take_one</code>   | Set to TRUE if the party that got the most votes in a district must get <i>at least</i> one seat ('Majorzbedingung') in this district. This only applies if the district winning party is entitled to a seat in the upper apportionment. Default is FALSE.                                                                                                                                                                    |
| <code>...</code>               | ignored (available for backwards compatibility)                                                                                                                                                                                                                                                                                                                                                                               |

**Details**

Each party nominates a candidate list for every district. The voters vote for the parties of their district. The seat allocation is calculated in two steps:

1. In the so called **upper apportionment** the number of seats for each party (over all districts) is determined.
2. In the so called **lower apportionment** the seats are distributed to the regional party list respecting the results from the upper apportionment.

Parties failing to reach quorums cannot get seats. This function does not handle seat assignment to candidates.

If you want to use other apportion methods than Sainte-Laguë use [biproporz\(\)](#).

### Value

A data.frame like votes\_df with a new column denoting the number of seats per party and district. Party and district divisors stored in attributes (hidden from print, see [get\\_divisors\(\)](#)). An ungrouped tibble is returned if votes\_df is a tibble.

### See Also

This function calls [biproporz\(\)](#) after preparing the input data.

### Examples

```
# Zug 2018
votes_df = unique(zug2018[c("list_id", "entity_id", "list_votes")])
district_seats_df = unique(zug2018[c("entity_id", "election_mandates")])

seats_df = pukelsheim(votes_df,
                      district_seats_df,
                      quorum_any(any_district = 0.05, total = 0.03),
                      winner_take_one = TRUE)

head(seats_df)

# Finland 2019
finland19_result = pukelsheim(finland2019$votes_df,
                              finland2019$district_seats_df,
                              new_seats_col = "mandates",
                              weight_votes = FALSE)
tail(finland19_result[order(finland19_result$mandates),])
```

---

quorum\_functions

---

*Create quorum functions for biproportional apportionment*


---

### Description

quorum\_any() and quorum\_all() are used for the quorum parameter in [biproporz\(\)](#)/[pukelsheim\(\)](#) and help describe how quorums should be applied prior to seat distributions.

### Usage

```
quorum_all(any_district, total)
```

```
quorum_any(any_district, total)
```

## Arguments

|                           |                                                                                                                                                                                                                                                                                                |
|---------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>any_district</code> | Vote threshold a party must reach in <i>at least</i> one district. Used as share of total votes within a district if less than 1 otherwise as number of votes. Must be greater than 0. Uses <code>reached_quorum_any_district()</code> .                                                       |
| <code>total</code>        | Vote threshold a party must reach for all votes cast. Used as share of total votes if less than 1. Otherwise as number of votes. Note that votes are not weighted with <code>weight_votes_matrix()</code> across districts. Must be greater than 0. Uses <code>reached_quorum_total()</code> . |

## Details

There's a difference in how the functions work. With `quorum_any`, *at least one* quorum must be reached. With `quorum_all` *all* (i.e. both) quorums must be reached. If you only use one parameter, `quorum_any()` and `quorum_all()` are identical.

## Value

a function which, when called with `function(votes_matrix)`, returns a boolean vector with length equal to the number of lists/parties (`votes_matrix` rows). The vector shows whether a party has reached any/all quorums.

## See Also

`apply_quorum()` for standalone quorum calculations

## Examples

```
votes_matrix = matrix(c(502, 55, 80, 10, 104, 55, 0, 1), ncol = 2)
dimnames(votes_matrix) <- list(c("A", "B", "C", "D"), c("Z1", "Z2"))
seats = c(Z1 = 50, Z2 = 20)

# use as parameter in biproporz or pukelsheim (general use case)
biproporz(votes_matrix, seats,
          quorum = quorum_any(any_district = 0.1, total = 100))

biproporz(votes_matrix, seats,
          quorum = quorum_all(any_district = 0.1, total = 100))

biproporz(votes_matrix, seats, quorum = quorum_any(any_district = 0.1))

biproporz(votes_matrix, seats, quorum = quorum_any(total = 100))

biproporz(votes_matrix, seats, quorum = quorum_any(total = 0.5))

# the quorum parameter also accepts vectors (e.g. calculated elsewhere)
biproporz(votes_matrix, seats, quorum = c(FALSE, TRUE, TRUE, TRUE))
```

---

reached\_quorum\_any\_district

*Check if parties reached a quorum in at least one district*


---

### Description

Base implementation, used by [quorum\\_functions](#).

### Usage

```
reached_quorum_any_district(votes_matrix, quorum_districts)
```

### Arguments

votes\_matrix    votes matrix

quorum\_districts

Vote threshold a party must reach in *at least* one district. Used as fraction of total votes within a district if less than 1, otherwise as number of votes. Must be greater than 0.

### Value

Logical vector with length equal to the number of lists/parties (votes\_matrix rows) showing whether they reached the quorum or not.

### See Also

[reached\\_quorum\\_total\(\)](#)

### Examples

```
(vm = matrix(c(239, 10, 308, 398, 20, 925), nrow = 3))
reached_quorum_any_district(vm, 25)
```

---

reached\_quorum\_total    *Check if parties reached the quorum for all votes*


---

### Description

Base implementation, used by [quorum\\_functions](#).

### Usage

```
reached_quorum_total(votes_matrix, quorum_total)
```

**Arguments**

|              |                                                                                                                                                             |
|--------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------|
| votes_matrix | votes matrix                                                                                                                                                |
| quorum_total | Vote threshold a party must reach for all votes cast. Used as fraction of total votes if less than 1, otherwise as number of votes. Must be greater than 0. |

**Value**

Logical vector with length equal to the number of lists/parties (votes\_matrix rows) showing whether they reached the quorum or not.

**Note**

Votes are not weighted across districts. This is relevant if the quorum threshold is the minimal number of *voters* (either as percentage or absolute value). In this case, use [weight\\_votes\\_matrix\(\)](#) before calculating the quorum.

**See Also**

[reached\\_quorum\\_any\\_district\(\)](#)

**Examples**

```
(vm = matrix(c(239, 10, 308, 398, 20, 925), nrow = 3))
reached_quorum_total(vm, 35)
```

---

run\_app

---

*Use biproportional apportionment interactively in a shiny app*


---

**Description**

Use biproportional apportionment interactively in a shiny app

**Usage**

```
run_app(votes_matrix = NULL, district_seats = NULL)
```

**Arguments**

|                |                                            |
|----------------|--------------------------------------------|
| votes_matrix   | optional votes_matrix to load upon start   |
| district_seats | optional district_seats to load upon start |

**Value**

Calling the function starts the shiny app

## Examples

```
if(interactive()){
  # You need to have the packages 'shiny' and 'shinyMatrix' installed to run the app
  run_app()

  # It's possible to load a matrix with the app
  run_app(uri2020$votes_matrix, uri2020$seats_vector)
}
```

---

|                     |                            |
|---------------------|----------------------------|
| upper_apportionment | <i>Upper apportionment</i> |
|---------------------|----------------------------|

---

## Description

In the first step of biproportional apportionment parties are given seats according to the sum of their votes across all districts.

## Usage

```
upper_apportionment(
  votes_matrix,
  district_seats,
  weight_votes = TRUE,
  method = "round"
)
```

## Arguments

|                |                                                                                                                                                                                                                                                                                                                                                                                             |
|----------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| votes_matrix   | Vote count matrix with votes by party in rows and votes by district in columns.                                                                                                                                                                                                                                                                                                             |
| district_seats | Vector defining the number of seats per district. Must be the same length as <code>ncol(votes_matrix)</code> . Values are name-matched to <code>votes_matrix</code> columns if both are named. If the number of seats per district should be calculated according to the number of votes (not the general use case), a single number for the total number of seats can be used.             |
| weight_votes   | By default (TRUE) it is assumed that each voter in a district has as many votes as there are seats in a district. Thus, votes are weighted according to the number of available district seats with <code>weight_votes_matrix()</code> . Set to FALSE if the argument <code>votes_matrix</code> shows the number of <i>voters</i> (e.g. because they can only cast one vote for one party). |
| method         | Apportion method that defines how seats are assigned, see <code>proporz()</code> . Default is the Sainte-Laguë/Webster method.                                                                                                                                                                                                                                                              |

## Value

A named list with district seats (for `votes_matrix` columns) and party seats (for rows).

**Note**

The results from the upper apportionment define the number of seats for each party and the number of seats for each district for the whole voting area. The lower apportionment will only determine where (i.e. which district) the party seats are allocated. Thus, after the upper apportionment is done, the final strength of a party/district within the parliament is definite.

**See Also**

`biproporz()`, `lower_apportionment()`

**Examples**

```
votes_matrix = matrix(c(123,912,312,45,714,255,815,414,215), nrow = 3)
district_seats = c(7,5,8)

upper_apportionment(votes_matrix, district_seats)
```

---

uri2020

*Election Data for the Cantonal Council of Uri (2020)*

---

**Description**

Example election data from the 2020 election for the cantonal council of Uri (Landrat) in Switzerland. The data has been extracted from the report "Landratswahlen 2020: Statistische Auswertung".

**Usage**

uri2020

**Format**

List containing:

- `votes_matrix` the number of votes for each party and district (4 rows, 4 columns)
- `seats_vector` with the number of seats per district (length 4)

**Source**

<https://www.ur.ch/abstimmungen/termine/9322>

---

|                     |                              |
|---------------------|------------------------------|
| weight_votes_matrix | Create weighted votes matrix |
|---------------------|------------------------------|

---

**Description**

Weight votes by dividing the votes matrix entries by the number of seats per district. This method is used in [upper\\_apportionment\(\)](#) if weight\_votes is TRUE (default).

**Usage**

```
weight_votes_matrix(votes_matrix, district_seats)
```

```
weight_list_votes(votes_matrix, district_seats)
```

**Arguments**

votes\_matrix    votes matrix

district\_seats    seats per district, vector with same length as ncol(votes\_matrix) and names as colnames(votes\_matrix)

**Value**

the weighted votes\_matrix which contains the number of voters (not rounded)

**Note**

weight\_list\_votes() has been renamed to [weight\\_votes\\_matrix\(\)](#) in v1.5.2 and is deprecated.

**Examples**

```
weight_votes_matrix(uri2020$votes_matrix, uri2020$seats_vector)
```

---

|         |                                                      |
|---------|------------------------------------------------------|
| zug2018 | Election Data for the Cantonal Council of Zug (2018) |
|---------|------------------------------------------------------|

---

**Description**

Example election data from the 2018 election for the cantonal council of Zug (Kantonsrat) in Switzerland.

**Usage**

```
zug2018
```

**Format**

An object of class `data.frame` with 267 rows and 49 columns.

**Source**

Kanton Zug (01.07.2022, 10:27:58). Kantonsratswahl 2018 (CSV). <https://wab.zug.ch/elections/kantonsratswahl-2018/data-csv>

# Index

## \* data

finland2019, 8  
uri2020, 22  
zug2018, 23

apply\_quorum, 2  
apply\_quorum(), 18

biproporz, 3  
biproporz(), 2, 9, 12, 13, 17, 22

ceil\_at, 5

district\_winner\_matrix, 6  
district\_winner\_matrix(), 12  
divisor\_ceiling (divisor\_methods), 7  
divisor\_ceiling(), 7, 15  
divisor\_floor (divisor\_methods), 7  
divisor\_floor(), 7, 15  
divisor\_geometric (divisor\_methods), 7  
divisor\_geometric(), 7, 15  
divisor\_harmonic (divisor\_methods), 7  
divisor\_harmonic(), 7, 15  
divisor\_methods, 7  
divisor\_round (divisor\_methods), 7  
divisor\_round(), 7, 15

finland2019, 8

get\_divisors, 9  
get\_divisors(), 4, 17

highest\_averages\_method, 9  
highest\_averages\_method(), 7

largest\_remainder\_method, 10  
largest\_remainder\_method(), 15  
lower\_apportionment, 11  
lower\_apportionment(), 4, 22

pivot\_to\_df (pivot\_to\_matrix), 13

pivot\_to\_matrix, 13  
proporz, 14  
proporz(), 2, 4, 7, 11, 12, 21  
pukelsheim, 15  
pukelsheim(), 5, 9, 13, 17

quorum\_all (quorum\_functions), 17  
quorum\_all(), 4, 16  
quorum\_any (quorum\_functions), 17  
quorum\_any(), 4, 16  
quorum\_functions, 2, 3, 17, 19

reached\_quorum\_any\_district, 19  
reached\_quorum\_any\_district(), 18, 20  
reached\_quorum\_total, 19  
reached\_quorum\_total(), 18, 19  
run\_app, 20

upper\_apportionment, 21  
upper\_apportionment(), 11, 12, 23  
uri2020, 22

weight\_list\_votes  
    (weight\_votes\_matrix), 23  
weight\_votes\_matrix, 23  
weight\_votes\_matrix(), 4, 18, 20, 21, 23

zug2018, 23