

# Package ‘doseSens’

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**Title** Conduct Sensitivity Analysis with Continuous Exposures and Binary or Continuous Outcomes

**Version** 1.0.0

**Description** Performs sensitivity analysis for the sharp null, attributable effects, and weak nulls in matched studies with continuous exposures and binary or continuous outcomes as described in Zhang, Small, Heng (2024) <[doi:10.48550/arXiv.2401.06909](https://doi.org/10.48550/arXiv.2401.06909)> and Zhang, Heng (2024) <[doi:10.48550/arXiv.2401.06909](https://doi.org/10.48550/arXiv.2401.06909)>. Installations require installation of the 'Gurobi' optimizer. Please see <[https://docs.gurobi.com/current/#refman/ins\\_the\\_r\\_package.html](https://docs.gurobi.com/current/#refman/ins_the_r_package.html)> for guidance.

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## Contents

|                                       |   |
|---------------------------------------|---|
| apply_permutation_to_matrix . . . . . | 2 |
| binary_thresh_attribute . . . . .     | 3 |
| change_Delta . . . . .                | 4 |
| constant_effects_test . . . . .       | 5 |
| dev_TV . . . . .                      | 6 |
| dose_attributable_general . . . . .   | 7 |
| dose_sensitivity_mc_gen . . . . .     | 9 |

|                                               |           |
|-----------------------------------------------|-----------|
| dose_thresh_attributable_one_sided . . . . .  | 11        |
| extract_above_threshold_vs_baseline . . . . . | 13        |
| extract_below_threshold_vs_baseline . . . . . | 14        |
| extract_max_vs_baseline . . . . .             | 14        |
| extract_min_vs_baseline . . . . .             | 15        |
| extract_OLS . . . . .                         | 16        |
| extract_stochastic_intervention . . . . .     | 16        |
| extract_threshold_effect . . . . .            | 17        |
| extract_threshold_effect_function . . . . .   | 18        |
| lead_bmd . . . . .                            | 18        |
| lead_crime . . . . .                          | 19        |
| max_expectation . . . . .                     | 19        |
| max_ratio . . . . .                           | 20        |
| max_ratios_summary . . . . .                  | 21        |
| max_ratio_new . . . . .                       | 21        |
| normal_test_gen . . . . .                     | 22        |
| prob_bounds . . . . .                         | 23        |
| sharp_double_statistic . . . . .              | 24        |
| sharp_null_double_test . . . . .              | 24        |
| var_est . . . . .                             | 26        |
| weak_null_test . . . . .                      | 26        |
| <b>Index</b>                                  | <b>29</b> |

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|                                                                                                |
|------------------------------------------------------------------------------------------------|
| apply_permutation_to_matrix                                                                    |
| <i>A function that returns a permuted vector according to a matrix and permutation vector.</i> |

---

**Description**

A function that returns a permuted vector according to a matrix and permutation vector.

**Usage**

apply\_permutation\_to\_matrix(M, p)

**Arguments**

- M                    a square matrix of dimension n
- p                    a permuted version of the vector from 1:n

**Value**

a length n vector with ith entry corresponding to the i, p\_i entry in M

**Examples**

```
mat <- matrix(1:9, nrow = 3, ncol = 3)
perm <- c(2, 1, 3)
permuted <- apply_permutation_to_matrix(M = mat, p = perm)
```

---

binary\_thresh\_attribute

*Separable algorithm for threshold attributable effect in a sensitivity analysis with at most one over-exposed unit in each matched set. For a greater than alternative, finds the 'a' matched sets that most decrease the mean and/or variance.*

---

**Description**

Separable algorithm for threshold attributable effect in a sensitivity analysis with at most one over-exposed unit in each matched set. For a greater than alternative, finds the 'a' matched sets that most decrease the mean and/or variance.

**Usage**

```
binary_thresh_attribute(
  Z,
  Q,
  index,
  gamma,
  thresh = 0,
  a = 1,
  trans = identity,
  mc = 50000
)
```

**Arguments**

|        |                                                                                                  |
|--------|--------------------------------------------------------------------------------------------------|
| Z      | A length N vector of (nonnegative) observed doses.                                               |
| Q      | A length N vector of observed binary outcomes.                                                   |
| index  | A length N vector of indices indicating matched set membership.                                  |
| gamma  | The nonnegative sensitivity parameter; gamma = 0 means no unmeasured confounding.                |
| thresh | The dose threshold for the TAE.                                                                  |
| a      | The number of attributable effects to test for.                                                  |
| trans  | The transformation of the doses to use for the test statistic. Default is the identity function. |
| mc     | Number of monte-carlo samples if testing the sharp null, i.e. a = 0.                             |

**Value**

Either "reject" if the value  $a$  is deemed not plausible/compatible, "feasible" if the value  $a$  is deemed so, else a list containing a p-value and dataframe of matched sets that have contribution to the test statistic sorted in order of smallest mean reduction followed by smallest variance reduction.

**Examples**

```
# Load the data
data <- lead_crime
# Solve by the separable algorithm
solution <- binary_thresh_attribute(data$log_lead, data$complain, data$matched_sets,
  gamma = 0, thresh = log(3.5), a = 5, trans = identity)
```

---

|              |                                                                                                                                                                                                                                                                                                                                                                                                                                  |
|--------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| change_Delta | <i>A helper that takes a gurobi model object outputted from dose_attributable_general or dose_thresh_attributable_one_sided and changes the Delta parameter. Saves computation time by directly editing the constraint involving Delta without having to reinitialize the other constraints that are kept the same. Outputs a list analogous to output from dose_attributable_general or dose_thresh_attributable_one_sided.</i> |
|--------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

---

**Description**

A helper that takes a gurobi model object outputted from `dose_attributable_general` or `dose_thresh_attributable_one_sided` and changes the Delta parameter. Saves computation time by directly editing the constraint involving Delta without having to reinitialize the other constraints that are kept the same. Outputs a list analogous to output from `dose_attributable_general` or `dose_thresh_attributable_one_sided`.

**Usage**

```
change_Delta(model, Delta, direction = "equal", TT)
```

**Arguments**

|           |                                                                               |
|-----------|-------------------------------------------------------------------------------|
| model     | A gurobi model object outputted from <code>dose_attributable_general</code> . |
| Delta     | The new Delta to test for.                                                    |
| direction | The new direction to test                                                     |
| TT        | The observed test statistic.                                                  |

**Value**

A gurobi model and solution.

---

constant\_effects\_test *Asymptotic sensitivity analysis for weak nulls with continuous exposures assuming constant effects across matched sets.*

---

## Description

Asymptotic sensitivity analysis for weak nulls with continuous exposures assuming constant effects across matched sets.

## Usage

```
constant_effects_test(
  Z,
  R,
  index,
  gamma = 0,
  theta = 0,
  X = NA,
  estimand_function = extract_OLS,
  gamma_star_vec = NULL
)
```

## Arguments

|                   |                                                                                                                     |
|-------------------|---------------------------------------------------------------------------------------------------------------------|
| Z                 | A length N vector of observed doses.                                                                                |
| R                 | A length N vector of observed outcomes.                                                                             |
| index             | A length N vector of indices indicating matched set membership.                                                     |
| gamma             | The nonnegative sensitivity parameter; gamma = 0 means no unmeasured confounding.                                   |
| theta             | The value at which to test the weak null.                                                                           |
| X                 | A matrix with I rows and less than I columns that contains covariate information.                                   |
| estimand_function | A function that takes in values z and r and outputs a scalar; this function governs the causal estimand to estimate |
| gamma_star_vec    | that contains the maximum ratio of any two probabilities of permutations for each matched set.                      |

## Value

A list containing the deviate, one-sided p-value, observed value of the test statistic in each matched set, and conservative standard deviation estimate.

## Examples

```
# Load the data
data <- lead_bmd
# prepare data
threshold <- log(0.74675)
match_info = data |> dplyr::group_by(matched_sets) |>
dplyr::summarise(below = sum(log_lead < threshold) > 0, disc = var(log_lead) > 0,
above = sum(log_lead > threshold) > 0)
below_indices <- match_info$matched_sets[match_info$below]
disc_indices <- match_info$matched_sets[match_info$disc]
above_indices <- match_info$matched_sets[match_info$above]
# outcome analysis using the stochastic intervention statistic, weak null
below_nbp <- data |> dplyr::filter(matched_sets %in% below_indices & matched_sets
%in% disc_indices)
above_below <- below_nbp |> dplyr::filter(matched_sets %in% above_indices)
extract_below_threshold_vs_baseline_function <- function(z, r) {
  extract_below_threshold_vs_baseline(z, r, threshold)
}
# one-sided test that estimand defined by estimand_function is 0 at gamma = 0
result <- constant_effects_test(Z = above_below$log_lead,
R = above_below$lumbar_spine_bmd,
index = above_below$matched_sets, gamma = 0, theta = 0,
estimand_function = extract_below_threshold_vs_baseline_function)
```

---

dev\_TV

*Computes deviation from uniform distribution in total variation distance for a given amount of unmeasured confounding and a greater than alternative with a binary outcome.*

---

## Description

Computes deviation from uniform distribution in total variation distance for a given amount of unmeasured confounding and a greater than alternative with a binary outcome.

## Usage

```
dev_TV(Z, Q, index, gamma, direct = "upper")
```

## Arguments

|        |                                                                                   |
|--------|-----------------------------------------------------------------------------------|
| Z      | A length N vector of (nonnegative) observed doses.                                |
| Q      | A length N vector of observed binary outcomes.                                    |
| index  | A length N vector of indices indicating matched set membership.                   |
| gamma  | The nonnegative sensitivity parameter; gamma = 0 means no unmeasured confounding. |
| direct | The direction of the test - "upper" or "lower"; default is upper.                 |

**Value**

A vector of length equaling the number of matched sets consisting of the TV distance from the uniform for each matched set at gamma level of unmeasured confounding for the worst-case.

**Examples**

```
# Load the data
data <- lead_crime
# compute total variation distances.
total_variation <- dev_TV(data$log_lead, data$complain,
data$matched_sets, gamma = log(1.5))
```

---

dose\_attributable\_general

*Inference for general attributable effects in sensitivity analysis with continuous exposures and binary outcomes. Gurobi must be installed to use this function.*

---

**Description**

Inference for general attributable effects in sensitivity analysis with continuous exposures and binary outcomes. Gurobi must be installed to use this function.

**Usage**

```
dose_attributable_general(
  Z,
  Q,
  index,
  gamma,
  alpha = 0.05,
  monotone = TRUE,
  Delta,
  sign = "positive",
  trans = identity,
  direction = "equal",
  M = 10000,
  eps = 0.01,
  alternative = "both",
  mv_threshold = NA,
  baseline = 0,
  relax = FALSE,
  feasible = TRUE,
  MIPgap = 0.01,
  WorkLimit = 1000,
  OutputFlag = 0
)
```

**Arguments**

|              |                                                                                                                                                                                     |
|--------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Z            | A length N vector of (nonnegative) observed doses.                                                                                                                                  |
| Q            | A length N vector of observed binary outcomes.                                                                                                                                      |
| index        | A length N vector of indices indicating matched set membership.                                                                                                                     |
| gamma        | The nonnegative sensitivity parameter; $\gamma = 0$ means no unmeasured confounding.                                                                                                |
| alpha        | Level of the test.                                                                                                                                                                  |
| monotone     | Whether to impose a monotonicity constraint on the potential outcomes                                                                                                               |
| Delta        | A numeric for the attributable effect to be tested for.                                                                                                                             |
| sign         | The sign of monotonicity - "positive" or "negative".                                                                                                                                |
| trans        | The transformation of the doses to use for the test statistic. Default is the identity function.                                                                                    |
| direction    | A string indicating the direction of testing the attributable effect with respect to Delta; "greater", "equal", or "less"                                                           |
| M            | The numeric penalty parameter to ensure correct sign relationship.                                                                                                                  |
| eps          | precision parameter for the objective function if solving feasible = "Yes"                                                                                                          |
| alternative  | For constraining the rejection region using the test statistic with baseline potential outcomes, should it be constructed with "lower" alternative, "upper" alternative, or "both." |
| mv_threshold | The number of allowed monotonicity violations.                                                                                                                                      |
| baseline     | The baseline dose level for defining the threshold attributable effect.                                                                                                             |
| relax        | Whether to solve the continuous relaxation.                                                                                                                                         |
| feasible     | Whether to look for a feasible solution or to find the optimal solution.                                                                                                            |
| MIPgap       | Gurobi parameter specifying the precision of the mixed integer program solution; default is 0.01.                                                                                   |
| WorkLimit    | Gurobi parameter specifying the maximum work units before stopping; default is 1000.                                                                                                |
| OutputFlag   | 0 if Gurobi print output is desired, 1 otherwise; default 0.                                                                                                                        |

**Value**

A list containing the following:

|          |                                                                                                                                                                                                                        |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| sol      | A gurobi object containing the solution to the optimization. If feasible is TRUE, then failing to find a solution indicates rejection. If feasible is FALSE, then optimal value greater than zero indicates rejection. |
| attribut | The attributable effect attained by the least rejectable allocation of potential outcomes and unmeasured confounders.                                                                                                  |
| search   | A list of length the number of matched sets containing the matrix of compatible baseline potential outcomes in each matched set.                                                                                       |
| null_exp | The null expectation of the pivot attained by the least rejectable allocation of potential outcomes and unmeasured confounders.                                                                                        |



|           |                                                                                                                                                                     |
|-----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| obsT      | The value of the pivot attained by the least rejectable allocation of potential outcomes and unmeasured confounders.                                                |
| nps       | A vector with length the number of matched sets containing the size of each matched set.                                                                            |
| start.ind | A vector with length the number of matched sets containing the starting index of the decision variables pertaining to each matched set in the optimization program. |
| exp_upper | The expectation of the pivot attained by the unmeasured confounders equalling the baseline potential outcome.                                                       |
| exp_lower | The expectation of the pivot attained by the unmeasured confounders equalling 1 minus the baseline potential outcome.                                               |
| var_upper | The variance of the pivot attained by the unmeasured confounders equalling the baseline potential outcome.                                                          |
| var_lower | The variance of the pivot attained by the unmeasured confounders equalling 1 minus the baseline potential outcome.                                                  |
| dose      | A list of length the number of matched sets containing the vector of doses observed in each matched set.                                                            |
| model     | The initialized gurobi model.                                                                                                                                       |

### Examples

```
# To run the following example, Gurobi must be installed.
# Load the data
data <- lead_crime
# Make a threshold at log(3.5) transformation function.
above = function(Z) { return(Z > log(3.5)) }
# Solve the mixed-integer program.

solution = tryCatch(dose_attributable_general(data$log_lead,
data$complain, data$matched_sets, gamma=log(1),
alpha = 0.1, monotone = TRUE, trans = above,
Delta = 5, direction = "less", M = 10000, eps = 0.0001,
alternative = "upper", relax = FALSE, feasible = FALSE),
error = function(e) NULL
)
```

---

dose\_sensitivity\_mc\_gen

*Sharp null monte-carlo sensitivity analysis for continuous exposures and binary outcomes.*

---

### Description

Sharp null monte-carlo sensitivity analysis for continuous exposures and binary outcomes.

**Usage**

```
dose_sensitivity_mc_gen(
  Z,
  Q,
  index,
  mc,
  gamma,
  weights = NA,
  obsT = NULL,
  trans = identity,
  direct = "upper",
  seed = 1,
  verbose = FALSE
)
```

**Arguments**

|         |                                                                                                  |
|---------|--------------------------------------------------------------------------------------------------|
| Z       | A length N vector of (nonnegative) observed doses.                                               |
| Q       | A length N vector of observed binary outcomes.                                                   |
| index   | A length N vector of indices indicating matched set membership.                                  |
| mc      | An integer for the total number of Monte-Carlo samples desired.                                  |
| gamma   | The nonnegative sensitivity parameter; gamma = 0 means no unmeasured confounding.                |
| weights | Weights for each stratum to apply for the test statistic                                         |
| obsT    | The observed value of the test statistic; default is NULL                                        |
| trans   | The transformation of the doses to use for the test statistic. Default is the identity function. |
| direct  | The direction of the test - "upper" or "lower"; default is upper.                                |
| seed    | seed for random number generation.                                                               |
| verbose | Whether to print status updates or not; default is FALSE.                                        |

**Value**

A list containing two objects:

|    |                                                                              |
|----|------------------------------------------------------------------------------|
| mc | A length mc vector containing the monte-carlo samples of the test statistic. |
| p  | The monte-carlo p-value.                                                     |

**Examples**

```
# Load the data
data <- lead_crime
# Make a threshold at log(3.5) transformation function.
above = function(Z) { return(Z > log(3.5)) }
# Conduct randomization test.
solution <- dose_sensitivity_mc_gen(data$log_lead, data$complain, data$matched_sets,
```

```
mc = 250, gamma = 0, trans = above)
```

---

```
dose_thresh_attributable_one_sided
```

*Inference for threshold attributable effects in sensitivity analysis with continuous exposures and binary outcomes. Gurobi must be installed to use this function.*

---

## Description

Inference for threshold attributable effects in sensitivity analysis with continuous exposures and binary outcomes. Gurobi must be installed to use this function.

## Usage

```
dose_thresh_attributable_one_sided(
  Z,
  Q,
  index,
  gamma,
  alpha = 0.05,
  monotone = TRUE,
  Delta,
  sign = "positive",
  direction = "equal",
  threshold = 0,
  M = 10000,
  eps = 0.01,
  mv_threshold = NA,
  baseline = 0,
  relax = FALSE,
  feasible = TRUE,
  MIPgap = 0.01,
  WorkLimit = 1000,
  OutputFlag = 0
)
```

## Arguments

|       |                                                                                   |
|-------|-----------------------------------------------------------------------------------|
| Z     | A length N vector of (nonnegative) observed doses.                                |
| Q     | A length N vector of observed binary outcomes.                                    |
| index | A length N vector of indices indicating matched set membership.                   |
| gamma | The nonnegative sensitivity parameter; gamma = 0 means no unmeasured confounding. |
| alpha | Level of the test.                                                                |

|              |                                                                                                                           |
|--------------|---------------------------------------------------------------------------------------------------------------------------|
| monotone     | Whether to impose a monotonicity constraint on the potential outcomes                                                     |
| Delta        | A numeric for the attributable effect to be tested for.                                                                   |
| sign         | The sign of monotonicity - "positive" or "negative".                                                                      |
| direction    | A string indicating the direction of testing the attributable effect with respect to Delta; "greater", "equal", or "less" |
| threshold    | The threshold for the TAE.                                                                                                |
| M            | The numeric penalty parameter to ensure correct sign relationship.                                                        |
| eps          | precision parameter for the objective function if solving feasible = "Yes"                                                |
| mv_threshold | The number of allowed monotonicity violations.                                                                            |
| baseline     | The baseline dose level for defining the threshold attributable effect.                                                   |
| relax        | Whether to solve the continuous relaxation.                                                                               |
| feasible     | Whether to look for a feasible solution or to find the optima.                                                            |
| MIPgap       | Gurobi parameter specifying the precision of the mixed integer program solution; default is 0.01.                         |
| WorkLimit    | Gurobi parameter specifying the maximum work units before stopping; default is 1000.                                      |
| OutputFlag   | 0 if Gurobi print output is desired, 1 otherwise; default 0.                                                              |

### Value

A list containing the following:

|           |                                                                                                                                                                                                                        |
|-----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| sol       | A gurobi object containing the solution to the optimization. If feasible is TRUE, then failing to find a solution indicates rejection. If feasible is FALSE, then optimal value greater than zero indicates rejection. |
| attribut  | The attributable effect attained by the least rejectable allocation of potential outcomes and unmeasured confounders.                                                                                                  |
| search    | A list of length the number of matched sets containing the matrix of compatible baseline potential outcomes in each matched set.                                                                                       |
| null_exp  | The null expectation of the pivot attained by the least rejectable allocation of potential outcomes and unmeasured confounders.                                                                                        |
| obsT      | The value of the pivot attained by the least rejectable allocation of potential outcomes and unmeasured confounders.                                                                                                   |
| nps       | A vector with length the number of matched sets containing the size of each matched set.                                                                                                                               |
| start.ind | A vector with length the number of matched sets containing the starting index of the decision variables pertaining to each matched set in the optimization program.                                                    |
| exp_upper | The expectation of the pivot attained by the unmeasured confounders equalling the baseline potential outcome.                                                                                                          |
| var_upper | The variance of the pivot attained by the unmeasured confounders equalling the baseline potential outcome.                                                                                                             |
| dose      | A list of length the number of matched sets containing the vector of doses observed in each matched set.                                                                                                               |
| model     | The initialized gurobi model.                                                                                                                                                                                          |

**Examples**

```
# To run the following example, Gurobi must be installed.
# Load the data
data <- lead_crime
# Solve the mixed-integer program.

solution = tryCatch(dose_thresh_attributable_one_sided(data$log_lead,
data$complain, data$matched_sets,
gamma=log(1), alpha = 0.1, monotone = TRUE, Delta = 5,
direction = "less", threshold = log(3.5), M = 10000,
eps = 0.0001, relax = FALSE, feasible = FALSE),
error = function(e) NULL
)
```

---

```
extract_above_threshold_vs_baseline
```

*Compute average of outcomes above dose threshold minus average of outcomes.*

---

**Description**

Compute average of outcomes above dose threshold minus average of outcomes.

**Usage**

```
extract_above_threshold_vs_baseline(z, r, threshold)
```

**Arguments**

|           |                      |
|-----------|----------------------|
| z         | a vector of doses    |
| r         | a vector of outcomes |
| threshold | a dose threshold     |

**Value**

the average of the outcomes with dose z above threshold c minus the average of the outcomes r.

**Examples**

```
# dose vector
dose <- c(0, 0.1, 0.4)
# outcome vector
outcome <- c(1, 1.1, 1.5)
theta <- extract_above_threshold_vs_baseline(z = dose, r = outcome, threshold = 0.3)
```

---

```
extract_below_threshold_vs_baseline
```

*Compute average of outcomes below dose threshold minus average of outcomes.*

---

### Description

Compute average of outcomes below dose threshold minus average of outcomes.

### Usage

```
extract_below_threshold_vs_baseline(z, r, threshold)
```

### Arguments

|           |                      |
|-----------|----------------------|
| z         | a vector of doses    |
| r         | a vector of outcomes |
| threshold | a dose threshold     |

### Value

the average of the outcomes with dose z below threshold c minus the average of the outcomes r.

### Examples

```
# dose vector
dose <- c(0, 0.1, 0.4)
# outcome vector
outcome <- c(1, 1.1, 1.5)
theta <- extract_below_threshold_vs_baseline(z = dose, r = outcome, threshold = 0.3)
```

---

```
extract_max_vs_baseline
```

*Compute largest dose outcome minus average of other outcomes.*

---

### Description

Compute largest dose outcome minus average of other outcomes.

### Usage

```
extract_max_vs_baseline(z, r)
```

### Arguments

|   |                      |
|---|----------------------|
| z | a vector of doses    |
| r | a vector of outcomes |

**Value**

the outcome  $r$  corresponding to the largest dose  $z$  minus the average of the outcomes  $r$ .

**Examples**

```
# dose vector
dose <- c(0, 0.1, 0.4)
# outcome vector
outcome <- c(1, 1.1, 1.5)
theta <- extract_max_vs_baseline(z = dose, r = outcome)
```

---

`extract_min_vs_baseline`

*Compute smallest dose outcome minus average of other outcomes.*

---

**Description**

Compute smallest dose outcome minus average of other outcomes.

**Usage**

```
extract_min_vs_baseline(z, r)
```

**Arguments**

|     |                      |
|-----|----------------------|
| $z$ | a vector of doses    |
| $r$ | a vector of outcomes |

**Value**

the outcome  $r$  corresponding to the smallest dose  $z$  minus the average of the outcomes  $r$ .

**Examples**

```
# dose vector
dose <- c(0, 0.1, 0.4)
# outcome vector
outcome <- c(1, 1.1, 1.5)
theta <- extract_min_vs_baseline(z = dose, r = outcome)
```

---

|             |                                                                                                    |
|-------------|----------------------------------------------------------------------------------------------------|
| extract_OLS | <i>A function that returns the coefficient from regressing an outcome vector on a dose vector.</i> |
|-------------|----------------------------------------------------------------------------------------------------|

---

**Description**

A function that returns the coefficient from regressing an outcome vector on a dose vector.

**Usage**

```
extract_OLS(z, r)
```

**Arguments**

|   |                      |
|---|----------------------|
| z | a vector of doses    |
| r | a vector of outcomes |

**Value**

the OLS regression coefficient from regressing r on z.

**Examples**

```
# dose vector
dose <- c(0, 0.1, 0.4)
# outcome vector
outcome <- c(1, 1.1, 1.5)
beta <- extract_OLS(z = dose, r = outcome)
```

---

|                                 |                                          |
|---------------------------------|------------------------------------------|
| extract_stochastic_intervention | <i>Compute weighted sum of outcomes.</i> |
|---------------------------------|------------------------------------------|

---

**Description**

Compute weighted sum of outcomes.

**Usage**

```
extract_stochastic_intervention(z, r, s)
```

**Arguments**

|   |                                |
|---|--------------------------------|
| z | a vector of doses              |
| r | a vector of outcomes           |
| s | a set of weights, summing to 1 |



**Value**

the inner product of s and r

**Examples**

```
# dose vector
dose <- c(0, 0.1, 0.4)
# outcome vector
outcome <- c(1, 1.1, 1.5)
# weight vector
weight = c(0.3, 0.4, 0.3)
theta <- extract_stochastic_intervention(z = dose, r = outcome, s = weight)
```

---

extract\_threshold\_effect

*Compute difference in average outcomes above and below a dose threshold.*

---

**Description**

Compute difference in average outcomes above and below a dose threshold.

**Usage**

```
extract_threshold_effect(z, r, threshold)
```

**Arguments**

|           |                      |
|-----------|----------------------|
| z         | a vector of doses    |
| r         | a vector of outcomes |
| threshold | a dose threshold     |

**Value**

the average of the outcomes with dose z above threshold c minus the average of the outcomes with dose z below the threshold c.

**Examples**

```
# dose vector
dose <- c(0, 0.1, 0.4)
# outcome vector
outcome <- c(1, 1.1, 1.5)
theta <- extract_threshold_effect(z = dose, r = outcome, threshold = 0.3)
```

---

```
extract_threshold_effect_function
```

*Function factory for extract\_threshold\_effect.*

---

### Description

Function factory for extract\_threshold\_effect.

### Usage

```
extract_threshold_effect_function(threshold = 0)
```

### Arguments

threshold      a dose threshold

### Value

A function that corresponds to extract\_threshold\_effect with the given threshold

### Examples

```
threshold_function <- extract_threshold_effect_function(threshold = 0.3)
```

---

```
lead_bmd
```

*Matched lead bone mineral density dataset*

---

### Description

A matched, trimmed dataset of lead exposure and lumbar bone mineral density. The data comes from NHANES 2011-18. There are 711 matched sets.

### Usage

```
lead_bmd
```

### Format

lead\_bmd:

A data frame with 1,436 rows and 23 columns:

**log\_lead** The log of lead exposure level measured in micrograms per deciliter.

**lumbar\_spine\_bmd** Bone mineral density in the lumbar spine in g/cm<sup>2</sup>

**matched\_sets** Matched set membership. ...

### Source

<https://wwwn.cdc.gov/nchs/nhanes/default.aspx>

---

|            |                                   |
|------------|-----------------------------------|
| lead_crime | <i>Matched lead crime dataset</i> |
|------------|-----------------------------------|

---

### Description

A matched, trimmed dataset of early life lead exposure and juvenile delinquency from a public dataset. There are 2007 matched sets.

### Usage

```
lead_crime
```

### Format

lead\_crime:

A data frame with 4,134 rows and 17 columns:

**log\_lead** The log of lead exposure level measured in micrograms per deciliter.

**serious** Whether the juvenile committed a serious offense.

**complain** Whether the juvenile committed an offense worthy of complaint.

**matched\_sets** Matched set membership. ...

### Source

<https://scholarworks.iu.edu/dspace/handle/2022/25638>

---

|                 |                                                                                                            |
|-----------------|------------------------------------------------------------------------------------------------------------|
| max_expectation | <i>A function to compute a conservative upper bound on the worst-case expectation under the sharp null</i> |
|-----------------|------------------------------------------------------------------------------------------------------------|

---

### Description

A function to compute a conservative upper bound on the worst-case expectation under the sharp null

### Usage

```
max_expectation(z, gamma, f_pi, with_variance = FALSE)
```

**Arguments**

|               |                                                                                                                                                                                                                           |
|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| z             | vector of doses of length n                                                                                                                                                                                               |
| gamma         | The nonnegative sensitivity parameter; gamma = 0 means no unmeasured confounding.                                                                                                                                         |
| f_pi          | a vector of length n! that contains the value of the test statistic under each of the n! permutations of z, with order of f_pi determined by first sorting z into increasing order and calling gtools::permutations on z. |
| with_variance | whether to return the variance along with the worst-case expectation, default is FALSE.                                                                                                                                   |

**Value**

a list containing the worst-case expectation, and/or variance and the solution to the optimization problem.

**Examples**

```
# A vector of observed doses
doses <- c(0, 0.1, 0.4, 0.8)
# values of test statistic under 4! permutations
values <- c(1, 0.5, 0.3, 0.8, 1, 0.7)
upper_bound <- max_expectation(z = doses, gamma = 1, f_pi = values)
```

---

|           |                                                                      |
|-----------|----------------------------------------------------------------------|
| max_ratio | <i>Find the max ratio of probabilities between two permutations.</i> |
|-----------|----------------------------------------------------------------------|

---

**Description**

Find the max ratio of probabilities between two permutations.

**Usage**

```
max_ratio(z, gamma)
```

**Arguments**

|       |                                 |
|-------|---------------------------------|
| z     | vector of doses                 |
| gamma | level of unmeasured confounding |

**Value**

the maximum ratio between the probability of two different permutations under the Rosenbaum model with doses z and unmeasured confounding level gamma.

**Examples**

```
# A vector of observed doses
doses <- c(0, 0.1, 0.4, 0.8)
ratio <- max_ratio(z = doses, gamma = 1)
```

---

|                    |                                                                                           |
|--------------------|-------------------------------------------------------------------------------------------|
| max_ratios_summary | <i>Find the max ratio of probabilities between two permutations for each matched set.</i> |
|--------------------|-------------------------------------------------------------------------------------------|

---

**Description**

Find the max ratio of probabilities between two permutations for each matched set.

**Usage**

```
max_ratios_summary(Z, index, gamma)
```

**Arguments**

|       |                                                                                   |
|-------|-----------------------------------------------------------------------------------|
| Z     | A length N vector of observed doses.                                              |
| index | A length N vector of indices indicating matched set membership.                   |
| gamma | The nonnegative sensitivity parameter; gamma = 0 means no unmeasured confounding. |

**Value**

A vector of length equaling the number of unique indices that contains the maximum ratio between any two permutations for each of the matched sets.

**Examples**

```
# A vector of observed doses
doses <- c(0, 0.1, 0.4, 0.8, 1)
matched_set <- c(1, 1, 1, 2, 2)
ratios <- max_ratios_summary(Z = doses, index = matched_set, gamma = 1)
```

---

|               |                                                                      |
|---------------|----------------------------------------------------------------------|
| max_ratio_new | <i>Find the max ratio of probabilities between two permutations.</i> |
|---------------|----------------------------------------------------------------------|

---

**Description**

Find the max ratio of probabilities between two permutations.

**Usage**

```
max_ratio_new(z, gamma)
```

**Arguments**

|       |                                                                                   |
|-------|-----------------------------------------------------------------------------------|
| z     | vector of doses                                                                   |
| gamma | The nonnegative sensitivity parameter; gamma = 0 means no unmeasured confounding. |

**Value**

the maximum ratio between the probability of two different permutations under the Rosenbaum model with doses  $z$  and unmeasured confounding level  $\gamma$ .

**Examples**

```
# A vector of observed doses
doses <- c(0, 0.1, 0.4, 0.8)
ratio <- max_ratio_new(z = doses, gamma = 1)
```

---

|                 |                                                                                                                 |
|-----------------|-----------------------------------------------------------------------------------------------------------------|
| normal_test_gen | <i>Sharp null sensitivity analysis for continuous exposures and binary outcomes using normal approximation.</i> |
|-----------------|-----------------------------------------------------------------------------------------------------------------|

---

**Description**

Sharp null sensitivity analysis for continuous exposures and binary outcomes using normal approximation.

**Usage**

```
normal_test_gen(
  Z,
  Q,
  index,
  gamma,
  trans = identity,
  weights = NA,
  obsT = NULL,
  direct = "upper"
)
```

**Arguments**

|                |                                                                                                  |
|----------------|--------------------------------------------------------------------------------------------------|
| <b>Z</b>       | A length N vector of (nonnegative) observed doses.                                               |
| <b>Q</b>       | A length N vector of observed binary outcomes.                                                   |
| <b>index</b>   | A length N vector of indices indicating matched set membership.                                  |
| <b>gamma</b>   | The nonnegative sensitivity parameter; $\gamma = 0$ means no unmeasured confounding.             |
| <b>trans</b>   | The transformation of the doses to use for the test statistic. Default is the identity function. |
| <b>weights</b> | Weights to apply for the test statistic                                                          |
| <b>obsT</b>    | The observed value of the test statistic; default is NULL.                                       |
| <b>direct</b>  | The direction of the test - "upper" or "lower"; default is upper.                                |

**Value**

A list containing the following:

|         |                                          |
|---------|------------------------------------------|
| obsT    | The observed value of the test statistic |
| exp     | The worst-case expectation               |
| var     | The worst-case variance.                 |
| deviate | The normal approximation deviate.        |

**Examples**

```
# Load the data
data <- lead_crime
# Make a threshold at log(3.5) transformation function.
above = function(Z) { return(Z > log(3.5)) }
# Conduct randomization test using normal approximation.
solution <- normal_test_gen(data$log_lead, data$complain, data$matched_sets,
gamma = 0, trans = above)
```

---

|             |                                                                                 |
|-------------|---------------------------------------------------------------------------------|
| prob_bounds | <i>A function to find the maximum and minimum probability of a permutation.</i> |
|-------------|---------------------------------------------------------------------------------|

---

**Description**

A function to find the maximum and minimum probability of a permutation.

**Usage**

```
prob_bounds(z, gamma)
```

**Arguments**

|       |                                                                                   |
|-------|-----------------------------------------------------------------------------------|
| z     | vector of doses                                                                   |
| gamma | The nonnegative sensitivity parameter; gamma = 0 means no unmeasured confounding. |

**Value**

a list containing the maximum and minimum probability of a permutation under the Rosenbaum model with doses z and unmeasured confounding level gamma.

**Examples**

```
# A vector of observed doses
doses <- c(0, 0.1, 0.4, 0.8)
bounds <- prob_bounds(z = doses, gamma = 1)
```

---

sharp\_double\_statistic

*Statistic based on inner product between transformations of dose and outcome.*

---

### Description

Statistic based on inner product between transformations of dose and outcome.

### Usage

```
sharp_double_statistic(z, r, q1, q2)
```

### Arguments

|    |                                           |
|----|-------------------------------------------|
| z  | a vector of doses                         |
| r  | a vector of outcomes                      |
| q1 | a function that transforms the doses z    |
| q2 | a function that transforms the outcomes r |

### Value

a vector with values corresponding to the inner product of transformed by q1 permutations of z with transformed by q2 versions of r.

### Examples

```
# dose vector
dose <- c(0, 0.1, 0.4)
# outcome vector
outcome <- c(1, 1.1, 1.5)
# transforms
transform1 <- function(x) x
transform2 <- function(x) x
theta <- sharp_double_statistic(z = dose, r = outcome, q1 = transform1,
q2 = transform2)
```

---

sharp\_null\_double\_test

*Asymptotic sharp null sensitivity analysis for a class of test statistics accommodating continuous exposures and any scalar outcome.*

---

### Description

Asymptotic sharp null sensitivity analysis for a class of test statistics accommodating continuous exposures and any scalar outcome.



**Usage**

```
sharp_null_double_test(
  Z,
  R,
  index,
  gamma = 0,
  q1 = NA,
  q2 = NA,
  X = NA,
  stratum_weights = rep(NA, nostratum),
  conservative_variance = TRUE,
  double_rank = TRUE
)
```

**Arguments**

|                       |                                                                                   |
|-----------------------|-----------------------------------------------------------------------------------|
| Z                     | A length N vector of observed doses.                                              |
| R                     | A length N vector of observed outcomes.                                           |
| index                 | A length N vector of indices indicating matched set membership.                   |
| gamma                 | The nonnegative sensitivity parameter; gamma = 0 means no unmeasured confounding. |
| q1                    | A transformation to apply to the doses.                                           |
| q2                    | A transformation to apply to the outcomes                                         |
| X                     | A matrix with I rows and less than I columns that contains covariate information. |
| stratum_weights       | A weight vector.                                                                  |
| conservative_variance | Whether to use the conservative variance or not; default is TRUE.                 |
| double_rank           | Whether to use the ranks of the transformed doses and outcomes; default is TRUE.  |

**Value**

A list containing the deviate, one-sided p-value, observed value of the test statistic in each matched set, and conservative standard deviation estimate.

**Examples**

```
# Load the data
data <- lead_bmd
# conduct sharp null test at gamma = 0.
result <- sharp_null_double_test(Z = data$log_lead,
  R = -data$lumbar_spine_bmd, index = data$matched_sets, gamma = 0)
```

---

|         |                                           |
|---------|-------------------------------------------|
| var_est | <i>A function for variance estimation</i> |
|---------|-------------------------------------------|

---

**Description**

A function for variance estimation

**Usage**

```
var_est(y, W, H_Q)
```

**Arguments**

|     |                                                                                    |
|-----|------------------------------------------------------------------------------------|
| y   | a vector of length I containing the value of test statistics from each matched set |
| W   | weight vector of length I.                                                         |
| H_Q | hat matrix corresponding to a matrix Q with dimension I by L.                      |

**Value**

a conservative estimate of the standard deviation of the test statistic.

**Examples**

```
test_stat <- c(1, 2, 1.5)
weight <- rep(1, 3)
Q <- matrix(1:9, nrow = 3, ncol = 2)
hat <- Q %%% solve(t(Q) %%% Q) %%% t(Q)
```

---

|                |                                                                                  |
|----------------|----------------------------------------------------------------------------------|
| weak_null_test | <i>Asymptotic sensitivity analysis for weak nulls with continuous exposures.</i> |
|----------------|----------------------------------------------------------------------------------|

---

**Description**

Asymptotic sensitivity analysis for weak nulls with continuous exposures.

**Usage**

```
weak_null_test(
  Z,
  R,
  index,
  gamma = 0,
  theta = 0,
  X = NA,
  estimand_function = extract_OLS,
```

```

    gamma_star_vec = NULL,
    kappa_inv_vec = NULL
  )

```

### Arguments

|                   |                                                                                                                                            |
|-------------------|--------------------------------------------------------------------------------------------------------------------------------------------|
| Z                 | A length N vector of observed doses.                                                                                                       |
| R                 | A length N vector of observed outcomes.                                                                                                    |
| index             | A length N vector of indices indicating matched set membership.                                                                            |
| gamma             | The nonnegative sensitivity parameter; gamma = 0 means no unmeasured confounding.                                                          |
| theta             | The value at which to test the weak null.                                                                                                  |
| X                 | A matrix with I rows and less than I columns that contains covariate information.                                                          |
| estimand_function | A function that takes in values z and r and outputs a scalar; this function governs the causal estimand to estimate                        |
| gamma_star_vec    | A vector that contains the minimum probability of a permutation for each matched set; default is NULL.                                     |
| kappa_inv_vec     | A vector that contains the ratio of the maximum probability and minimum probability of a permutation for each matched set; default is NULL |

### Value

A list containing the deviate, one-sided p-value, observed value of the test statistic in each matched set, and conservative standard deviation estimate.

### Examples

```

# Load the data
data <- lead_bmd
# prepare data
threshold <- log(0.74675)
match_info = data |> dplyr::group_by(matched_sets) |>
dplyr::summarise(below = sum(log_lead < threshold) > 0, disc = var(log_lead) > 0,
above = sum(log_lead > threshold) > 0)
below_indices <- match_info$matched_sets[match_info$below]
disc_indices <- match_info$matched_sets[match_info$disc]
above_indices <- match_info$matched_sets[match_info$above]
# outcome analysis using the stochastic intervention statistic, weak null
below_nbp <- data |> dplyr::filter(matched_sets %in% below_indices & matched_sets
%in% disc_indices)
above_below <- below_nbp |> dplyr::filter(matched_sets %in% above_indices)
extract_below_threshold_vs_baseline_function <- function(z, r) {
  extract_below_threshold_vs_baseline(z, r, threshold)
}
# one-sided test that estimand defined by estimand_function is 0 at gamma = 0
result <- weak_null_test(Z = above_below$log_lead,
R = above_below$lumbar_spine_bmd,
index = above_below$matched_sets, gamma = 0, theta = 0,

```

```
estimand_function = extract_below_threshold_vs_baseline_function)
```

# Index

- \* **datasets**
  - lead\_bmd, [18](#)
  - lead\_crime, [19](#)
- apply\_permutation\_to\_matrix, [2](#)
- binary\_thresh\_attribute, [3](#)
- change\_Delta, [4](#)
- constant\_effects\_test, [5](#)
- dev\_TV, [6](#)
- dose\_attributable\_general, [7](#)
- dose\_sensitivity\_mc\_gen, [9](#)
- dose\_thresh\_attributable\_one\_sided, [11](#)
- extract\_above\_threshold\_vs\_baseline, [13](#)
- extract\_below\_threshold\_vs\_baseline, [14](#)
- extract\_max\_vs\_baseline, [14](#)
- extract\_min\_vs\_baseline, [15](#)
- extract\_OLS, [16](#)
- extract\_stochastic\_intervention, [16](#)
- extract\_threshold\_effect, [17](#)
- extract\_threshold\_effect\_function, [18](#)
- lead\_bmd, [18](#)
- lead\_crime, [19](#)
- max\_expectation, [19](#)
- max\_ratio, [20](#)
- max\_ratio\_new, [21](#)
- max\_ratios\_summary, [21](#)
- normal\_test\_gen, [22](#)
- prob\_bounds, [23](#)
- sharp\_double\_statistic, [24](#)
- sharp\_null\_double\_test, [24](#)
- var\_est, [26](#)
- weak\_null\_test, [26](#)